Violation of Bell inequalities through the coincidence-time loophole

Peter Morgan*
Physics Department, Yale University, New Haven, CT 06520, USA.†

The coincidence-time loophole was identified by Larsson & Gill (*Europhys. Lett.* **67**, 707 (2004)); a concrete model that exploits this loophole has recently been described by De Raedt *et al.* (*Found. Phys.*, to appear). It is emphasized here that De Raedt *et al.*'s model is experimentally testable. De Raedt *et al.*'s model also introduces contextuality in a novel and classically more natural way than the use of contextual particle properties, by introducing a probabilistic model of a limited set of degrees of freedom of the measurement apparatus, so that it can also be seen as a random field model. Even though De Raedt *et al.*'s model may well contradict detailed Physics, it nonetheless provides a way to simulate the logical operation of elements of a quantum computer, and may provide a way forward for more detailed random field models.

PACS numbers: 03.65.Ud

De Raedt *et al.*[1, 2, 3] (hereinafter RRMKM) construct a computer model that violates Bell-type inequalities, which can be used to simulate elements of a quantum computer at an event by event level. Although the RRMKM model can be understood as a computing simulation, *not* as a Physics model, it is a local model that can be said to exploit the "coincidence-time" loophole[4], which was identified by Larsson and Gill as "significantly more damaging than the well-studied detection problem"[5]. The RRMKM model is more concrete and less general than the models discussed in [5]. Such models would be of little interest to most Physicists were it not for the fact that the RRMKM model, if it is considered as a Physics model, is experimentally testable, and is a prototype for more detailed random field models.

The coincidence of events is part of the conventional definition of 2-particle states in quantum mechanics: if we observe two events at time-like separation, they may or may not be caused by the same particle; if we observe two events at space-like separation, they cannot be caused by a single particle, there must be a 2-particle state (or a 3-particle state, ...). For an archetypal experiment that measures a 2-particle state, we may turn to Weihs *et al.*'s measurement of a violation of Bell inequalities[6]. In this experiment, two computers recorded the times at which events occurred at each of the two ends of the experiment, then the two datasets were compared (on a third computer, although this is logically inessential) to determine when there were approximately matched events, "Coincidences were identified by calculating time differences between Alice's and Bob's time tags and comparing these with a time window (typically a few ns)"[6, p 5041]. The storage of two separate datasets followed by subsequent analysis is logically equivalent to a hardware coincidence circuit, but very usefully allows the retrospective analysis of the coincidences we would have observed if we had used different hardware coincidence circuits.

The RRMKM model can be understood on two levels: as a computer simulation of individual events; and as a probabilistic model that captures the properties of the simulation. The empirical adequacy of an event by event simulation model is established by comparison of statistics of the computer generated events with statistics of experiments, no reference to a probabilistic model is necessary, so a simulation approach to Physics is not necessarily parasitic on a probabilistic approach, but the presentation of computer simulations is not yet, to this author, transparent as Physics. Here, only the probabilistic approach will be discussed, because it is more appropriate as a conventional Physics model.

In brief, the RRMKM model understood in a probabilistic way depends on a probability density of the time at which a single event is observed, $p(t|\mathbf{a}.\mathbf{S})$, having a non-trivial dependence on the polarization of a light source[3, §6]. In quantum theory, a rotationally invariant 2-photon quantum state (which we will say — unrealistically, but for the sake of simplicity — has already been determined not to require helical polarization in its de-

*Electronic address: peter.w.morgan@yale.edu
†URL: http://pantheon.yale.edu/~PWM22

scription) is a mixture of a pure state,

$$\rho_p = \psi_p \psi_p^\dagger,\ \psi_p = \frac{1}{\sqrt{2}} \left(|H_{\mathbf{S}}\rangle_1 |V_{\mathbf{S}}\rangle_2 - |V_{\mathbf{S}}\rangle_1 |H_{\mathbf{S}}\rangle_2 \right), \quad (1)$$

which is invariant under rotation of the polarization vector $\mathbf{S}$, and a rotationally invariant mixed state,

$$\rho_m = \frac{1}{2\pi} \int d\mathbf{S}\, |H_{\mathbf{S}}\rangle_1 |V_{\mathbf{S}}\rangle_2 \langle V_{\mathbf{S}}|_2 \langle H_{\mathbf{S}}|_1 . \qquad (2)$$

Characterization of an optical source requires us to determine a range of such mixtures that model the source to a chosen empirical accuracy. A dependence of $p(t|\mathbf{a}.\mathbf{S})$ on the polarization of the light source, if observed, reduces our ability to limit the range of such states that are empirically adequate. One conclusion of this letter is therefore that the description of any experiment that depends on coincidences of events for different polarizations should include a characterization of the dependence of detector delay on different polarizations, because future experimentalists will have to reproduce that dependence to obtain the same results.

The RRMKM model understood in a probabilistic way works by constructing a familiar separable hidden variable model in terms of polarization vectors $\mathbf{S}_1$ and $\mathbf{S}_2$,

$$p(x_1, x_2, t_1, t_2|\mathbf{a}_1, \mathbf{a}_2) = \frac{1}{4\pi^2} \iint d\mathbf{S}_1 d\mathbf{S}_2 p(x_1|\mathbf{a}_1.\mathbf{S}_1) p(t_1|\mathbf{a}_1.\mathbf{S}_1) p(x_2|\mathbf{a}_2.\mathbf{S}_2) p(t_2|\mathbf{a}_2.\mathbf{S}_2) p(\mathbf{S}_1, \mathbf{S}_2), \qquad (3)$$

where $x_1, x_2 \in \{-1, +1\}$, depending on which detector triggers behind a polarizing filter aligned at angles $\mathbf{a}_1, \mathbf{a}_2$, respectively, at the two ends of the experiment. To model coincidence as it is described in the Weihs *et al.* experiment, we suppose that $|t_2 - t_1|$ must be less than a length of time W, and integrate over all time, to obtain

$$p(x_1, x_2|\mathbf{a}_1, \mathbf{a}_2) = \frac{\displaystyle\iint d\mathbf{S}_1 d\mathbf{S}_2 p(x_1|\mathbf{a}_1.\mathbf{S}_1) p(x_2|\mathbf{a}_2.\mathbf{S}_2) w(\mathbf{a}_1.\mathbf{S}_1, \mathbf{a}_2.\mathbf{S}_2, W) p(\mathbf{S}_1, \mathbf{S}_2)}{\displaystyle\iint d\mathbf{S}_1 d\mathbf{S}_2 w(\mathbf{a}_1.\mathbf{S}_1, \mathbf{a}_2.\mathbf{S}_2, W) p(\mathbf{S}_1, \mathbf{S}_2)}, \qquad (4)$$

where the weight function $w(\mathbf{a}_1.\mathbf{S}_1, \mathbf{a}_2.\mathbf{S}_2, W)$ is

$$\begin{aligned} w(\mathbf{a}_1.\mathbf{S}_1, \mathbf{a}_2.\mathbf{S}_2, W) &= \iint dt_1 dt_2 p(t_1|\mathbf{a}_1.\mathbf{S}_1) p(t_2|\mathbf{a}_2.\mathbf{S}_2) \Theta(W - |t_2 - t_1|) & (5) \\ &= 2W \int dt p(t|\mathbf{a}_1.\mathbf{S}_1) p(t|\mathbf{a}_2.\mathbf{S}_2) + O(W^2). & (6) \end{aligned}$$

Eq. (5) is an integral on a line of width $W\sqrt{2}$, centered on $t_1 = t_2$, leading to Eq. (6) when W is small. With an appropriate choice of $p(t|\mathbf{a}.\mathbf{S})$, $p(x_1, x_2|\mathbf{a}_1, \mathbf{a}_2)$ is not separable and may violate Bell inequalities[3, §6], so a local model can be constructed that reproduces the logical operation of a quantum computer (the logical operation of a quantum computer being independent of the detailed Physics, the usefulness of this approach will to some extent survive if further experiment invalidates them as Physics models). In particular, [3, §6] uses a pseudo-random model for the polarizer that reproduces Malus law, for which

$$p(x|\mathbf{a}.\mathbf{S}) = \frac{1-x}{2} + x(\mathbf{a}.\mathbf{S})^2 = \frac{1}{2}(1 + x\cos 2\zeta), \qquad (7)$$

where $\cos\zeta = \mathbf{a}.\mathbf{S}$, introduces a uniform distribution ansatz,

$$\begin{aligned} p(t|\mathbf{a}.\mathbf{S}) &= \frac{\Theta(t)\Theta(T(\mathbf{a}.\mathbf{S}) - t)}{T(\mathbf{a}.\mathbf{S})}, \\ \text{where } T(\mathbf{a}.\mathbf{S}) &= T_0 \left[4(\mathbf{a}.\mathbf{S})^2(1 - (\mathbf{a}.\mathbf{S})^2)\right]^{d/2} \\ &= T_0 \left|\sin 2\zeta\right|^d, \end{aligned} \qquad (8)$$

and chooses $p(\mathbf{S}_1, \mathbf{S}_2)$ so that $\mathbf{S}_1$ and $\mathbf{S}_2$ are orthogonal, to construct a model that matches the predictions of quantum theory for the pure rotationally invariant state of Eq. (1) when $d = 4$ and $W \ll T_0$ is small. When $d = 0$ or $W > T_0$ is large, the model satisfies Bell inequalities. The uniform distribution ansatz of this model is inessential to the violation of Bell inequalities, and the weight

function $w(\mathbf{a}_1.\mathbf{S}_1, \mathbf{a}_2.\mathbf{S}_2, W)$ does not determine $p(t|\mathbf{a}.\mathbf{S})$, so there is an infinite class of functions that violate Bell inequalities.

The RRMKM model is experimentally testable, provided we assume that after passing through a polarization filter the unobservable polarization vector $\mathbf{S}$ is aligned with the polarization filter (this assumption is required in conjunction with Eq.(7) to reproduce Malus law; the simplest quantum mechanical modeling of polarization filters is, comparably, as a projection of the state to the same alignment as the polarization filter). We can measure the delay dependency function $p(t|\mathbf{a}.\mathbf{S})$ for a given detector by directing a source of known polarization through a briefly open gate and observing whether there are timing differences for different relative orientations. Then we can compute the resulting weight function, $w(\mathbf{a}_1.\mathbf{S}_1, \mathbf{a}_2.\mathbf{S}_2, W)$, which establishes how much violation of the Bell inequality can be accounted for by detection delay. It is plausible from the classical optics of crystals that there will be such timing dependencies[7, §14.3.2,§14.4.1], but the extent of the timing dependencies required is considerable, since the violation of the Bell inequalities in Weihs *et al.*'s experiment only diminishes to zero as W is increased beyond 300ns[3, Fig. 3], corresponding to an effective path length difference of 100m. If the whole violation of the Bell inequalities by an experiment can be accounted for by the delay dependency of the detectors used, then there is a sense in which we have so far failed to characterize the state of the light source as definitely ρ_p, definitely ρ_m, or as one of the continuum of intermediate mixtures. Provided we consistently use detectors that have the same delay dependency and we use the same procedure to determine event coincidences, however, we can continue to use ρ_p to describe a light source (supposing that the measurement results — on the (false) assumption that we are using detectors for which $p(t|\mathbf{a}.\mathbf{S})$ is independent of $\mathbf{a}.\mathbf{S}$ — support ρ_p as a model), but if we use different detectors or different event coincidence criteria we must reassess the empirical effectiveness of ρ_p. If ρ_m successfully models an experiment when detectors are modelled accurately, this of course does not mean that an experiment *is* classical.

If detectors generally prove to have nontrivial dependencies of $p(t|\mathbf{a}.\mathbf{S})$ on $\mathbf{a}.\mathbf{S}$, we can retain a relatively straightforward conceptual position by insisting that an *ideal* quantum detector has no such dependency. This is a reasonable position to adopt even if all detectors have nontrivial dependencies on $\mathbf{a}.\mathbf{S}$, provided it can be proven that there is no limit to the reduction of such dependencies. Dependence of $p(t|\mathbf{a}.\mathbf{S})$ on $\mathbf{a}.\mathbf{S}$ in both the detectors is an *engineered* nonlocal correlation between the measurement apparatuses, because the detectors have the same internal structure. Together with the nonlocally defined determination of coincidence of events at space-like separation, this nonlocal correlation is enough to introduce a nondynamical nonlocality into this classical model. Seen in this way, the construction of the experimental apparatus is an example of what is pejoratively termed "conspiracy".

Modeling the dependence of $p(t|\mathbf{a}.\mathbf{S})$ on $\mathbf{a}.\mathbf{S}$ effectively introduces a small, critical part of the experimental apparatus into the experimental model, so we should also understand the model to be "contextual". The contextuality of the RRMKM model accommodates the Fine and Accardi discussion of Bell inequalities[8, 9, 10, 11, 12, 13], without, however, introducing contextual particle properties, which are rightly anathema to classical particle physics. Noting that the discussion of classical models for quantum mechanical systems has always stalled on whether the necessary modifications of classical physics are natural rather than whether they are possible, the RRMKM model moves classical models one more step towards naturalness. Details of the experimental apparatus have a subtle impact on how we model the experiment, making it increasingly difficult to understand the experiment as a "measurement" of "the system we are measuring". As we consider experiments in progressively more detail, we are forced to introduce more details of the experimental apparatus, so we cannot confine our quantum models to small numbers of electrons and photons, with the experimental apparatus represented only by a measurement operator. In effect, as we include more of the experimental apparatus, and increasingly finer details, we move the Heisenberg cut outwards into the world, making our experiments more and more capable of being modeled by classical random field methods[14, 15]. The significance of the RRMKM model is that it moves a *small* characteristic property of the measurement apparatus into our detailed discussion of the measurement, without requiring a perfect description of the whole mea-

surement device. Thanks to the arguments in [14], we know that if we move *enough* of the measurement apparatus into a model for an experiment, a classical model can violate Bell inequalities. Even if the RRMKM model is ruled out by measurement of $p(t|\mathbf{a}.\mathbf{S})$, nonetheless it gives a novel way to introduce contextuality into classical models, in a classically acceptable way, without introducing contextual particle properties.

Note that the above discussion is not affected by the critique of Hess and Philipp's discussion[16, 17] by Gill *et al.*[18, 19]. Here, timings and coincidences of events are explicitly at issue in a different way than highlighted in those papers, as noted by Larsson and Gill[5].

Quantum mechanics is not under threat as an engineering discipline — it is much too useful to separate the world into "the measurement apparatus" and "the system that is measured"; into "the measurement apparatus" and "the preparation apparatus"; or even into "the Universe" and "ideal quantum measurement"; all of which allow the mathematical tools of quantum theoretical measurement operators and Hilbert spaces of states to be used. This splitting of the world into two parts seems to be always possible For All Practical Purposes, *but* the choice is always pragmatic. This is of course the arbitrariness of the Heisenberg cut: the way in which we split the world into two parts is not a perfect truth about the world. This is a *fundamental* limitation of the mathematical tools of quantum theory. The immediate alternative, however, a classical holistic model that explicitly models quantum fluctuations[14, 15], is *no* better, since there cannot be a model of the whole universe: there is always a separation of the world into what is in the model and what is not in the model, so classical modeling is equally pragmatic. I therefore make no claims that we should construct classical models, but, pragmatically, it might sometimes be useful to do so, and we can better understand quantum (field) theory by better understanding the relationship between quantum and classical models. In particular, as quantum theoretical models increasingly introduce more details of the measurement apparatus, in a constant pursuit of more accuracy, contextual classical models will increasingly become alternatives of comparable complexity.

I am grateful to Hans De Raedt for correspondence and for clarifications of details of the RRMKM model.

---

[1] H. De Raedt, K. De Raedt, K. Michielsen, K. Keimpema, and S. Miyashita, J. Comput. Theor. Nanosci. **4**, 957 (2007); arXiv:0712.3781.

[2] H. De Raedt, K. De Raedt, K. Michielsen, K. Keimpema, and S. Miyashita, J. Phys. Soc. Jpn. **76**, 104005 (2007); arXiv:0712.2565.

[3] S. Zhao, H. De Raedt, and K. Michielsen, Found. Phys., to appear; arXiv:0712.3693

[4] M. P. Seevinck and J.-Å. Larsson, Eur. Phys. J. B **58**, 51 (2007).

[5] J.-Å. Larsson and R. D. Gill, Europhys. Lett. **67**, 707 (2004).

[6] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **81**, 5039(1998).

[7] M. Born and E. Wolf, "Principles of Optics" (5th Edn.), Pergamon, Oxford, UK, 1975.

[8] L. Accardi, Phys. Rep. **77**, 169 (1981).

[9] A. Fine, Phys. Rev. Lett. **48**, 291 (1982).

[10] W. M. de Muynck, Phys. Lett. A **114**, 65 (1986); W. M. de Muynck, "Foundations of Quantum Mechanics, an Empiricist Approach", Kluwer, Dordrecht, 2002.

[11] L. J. Landau, Phys. Lett. A **120**, 5456 (1987).

[12] R. F. Streater, J. Math. Phys. **41**, 3556 (2000).

[13] K. Hess and W. Philipp, "Connection of probability models to EPR experiments: Probability spaces and Bell's Theorem", in: Th. M. Nieuwenhuizen, V. Spicka, B. Mehmani, M. Jafar-Aghdami, and A. Yu. Khrennikov (eds.), 'Proceedings of "Beyond the Quantum", Leiden, 29th May – 2nd June, 2006', World Scientific, Singapore, 2007.

[14] J. Phys. A: Math. Gen. **39**, 7441 (2006); arXiv:cond-mat/0403692.

[15] J. Math. Phys. **48**, 122302 (2007); arXiv:0704.3420.

[16] K. Hess and W. Philipp, Proc. Natl. Acad. Sci. USA, **98**, 14224 (2001).

[17] K. Hess and W. Philipp, Proc. Natl. Acad. Sci. USA, **98**, 14228 (2001).

[18] R. D. Gill, G. Weihs, A. Zeilinger, and M.Żukowski, Proc. Natl. Acad. Sci. USA, **99**, 14632 (2002).

[19] R. D. Gill, G. Weihs, A. Zeilinger, and M.Żukowski, Europhys. Lett., **61**, 282 (2003).